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Polarization transfer in the d(p, n)2p reaction

G. V. DASS† and N. M. QUEEN‡

† Theoretical Physics Division, Atomic Energy Research Establishment, Harwell, Didcot, Berks.

‡ Department of Mathematical Physics, University of Birmingham

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Abstract. The impulse approximation is used to predict the depolarization parameter (transverse polarization transfer coefficient) in the d(p, n)2p reaction as a function of the energy of the neutrons emitted in the forward direction, for incident proton energies between 30 and 150 MeV. Multiple-scattering corrections are shown to be small at the high-energy end of the final neutron spectrum, where the formalism is most reliable. The depolarization parameter for these high-energy neutrons is found to be negative over the entire range of incident proton energies considered, and increases in magnitude as a function of this energy. However, the value of this parameter is particularly sensitive to the nucleon–nucleon phase shifts assumed in the calculation, so that its experimental determination would make it possible to discriminate between alternative sets of nucleon–nucleon phase shifts which fit all the two-body scattering data.

1. Introduction

In a recent experiment reported by Devins *et al.* (1966) a deuterium target was bombarded by transversely polarized protons of 30 and 50 MeV in an attempt to measure the transverse polarization of the neutrons emitted in the forward direction as a function of their energy. The primary motivation for their work was the possibility, first suggested by Phillips (1959), that polarization transfer in the d(p, n)2p reaction might be a good source of nearly monoenergetic polarized neutrons. Moreover, analysis of this reaction may be used to provide information on the neutron–proton interaction which is not readily available from two-body experiments or, alternatively, if the nucleon–nucleon interactions are accurately known at a particular energy, to test the reliability of theoretical techniques for handling the three-body problem.

Energy spectra of the outgoing neutrons in the reaction d(p, n)2p, and of the protons in its mirror reaction d(n, p)2n, have been predicted by several authors (Castillejo and Singh 1960, Phillips 1964) according to the impulse approximation. In each case there is a strong peak of high-energy outgoing nucleons. In addition, Phillips (1959) has estimated the depolarization parameter (transverse polarization transfer coefficient) D in the d(p, n)2p reaction, for fast neutrons emitted near the forward direction. He pointed out that the S state is strongly favoured for the residual low-energy proton pair, so that these protons are restricted to the singlet spin state by the Pauli principle. It is therefore sufficient to consider only transitions which leave the two protons in the singlet state. Although one of the protons may be regarded as a 'spectator' in this reaction, while the other two nucleons undergo a quasi-free charge-exchange process, the polarization transfer may be considerably different from that in free neutron–proton charge-exchange scattering, as a result of the restriction on the final spin states. Thus measurements of the D parameter for this process may be capable of distinguishing between alternative sets of nucleon–nucleon phase shifts which fit all the two-body scattering data equally well.

Near the low-energy end of the final neutron spectrum, where the proton pair has a large relative momentum, both singlet and triplet pp spin states are allowed. One would therefore expect that, for sufficiently large incident proton energies, the polarization transfer in the d(p, n)2p reaction reduces to that for free neutron–proton charge-exchange scattering.

The aim of the present paper is to give impulse-approximation predictions of the D parameter for the d(p, n)2p reaction over a range of energies, to estimate the effects of final-state interactions between the two protons and of multiple-scattering corrections to

the impulse approximation, and to determine the sensitivity of the results to the nucleon–nucleon phase shifts assumed in the analysis. The application of the impulse approximation to the high-energy end of the neutron spectrum is straightforward and unambiguous. However, for final states involving low-energy neutrons, some of the conventional approximations made in models based on the impulse approximation are highly questionable. We shall therefore study the high-energy end of the neutron spectrum in greater detail than the region of lower energies.

2. The impulse approximation

The d(p, n)2p scattering amplitude, regarded as a matrix in spin space, is proportional to the matrix element

$$\mathscr{M}_{ij} = \langle \mathbf{k}', \phi_{\mathbf{q}}, \chi_i, I_1 | \mathscr{T} | \mathbf{k}, \phi_0, \chi_j, I_0 \rangle \tag{1}$$

where $\mathscr{T}$ is the transition operator for the process, $\hbar\mathbf{k}$ and $\hbar\mathbf{k}'$ are the centre-of-mass momenta of the initial proton and final neutron respectively, ϕ_0 is the ground state of the deuteron, $\phi_{\mathbf{q}}$ is the wave function of the final pp system with relative momentum $\hbar\mathbf{q}$, the χ_i are three-nucleon spin states in some convenient orthonormal basis and

$$I_0 = p_1 \left(\frac{n_2 p_3 - p_2 n_3}{\sqrt{2}} \right) \tag{2}$$

$$I_1 = n_1 p_2 p_3 \tag{3}$$

are the initial and final isospin states, where the particle label 1 refers to the incident proton or final neutron, while 2 and 3 refer to the remaining nucleons. In the states I_0 and I_1 nucleons 2 and 3 have total isospin $I = 0$ and $I = 1$ respectively. The use of the isospin formalism here has the advantage that the two nucleons comprising the deuteron are treated on an equal footing. We take χ_i to be the spin states which are simultaneously eigenstates of the square of the total spin of all three nucleons and its z component (with the axis of quantization parallel to the vector $\mathbf{k}$), as well as the square of the total spin of nucleons 2 and 3. The small D-state component of the deuteron wave function is neglected.

For a given $\mathbf{k}'$ the magnitude of $\mathbf{q}$ is constrained by the energy conservation condition

$$\frac{3\hbar^2}{4M} k'^2 + \frac{\hbar^2}{M} q^2 = \frac{\hbar^2}{2\mu} k^2 + E_0 \tag{4}$$

where M is the nucleon mass, μ is the reduced mass of the nucleon–deuteron system and $E_0 = -2{\cdot}23$ MeV is the deuteron binding energy.

The single-scattering approximation is

$$\mathscr{T} \simeq T_2 + T_3 \tag{5}$$

where T_n is the transition operator for the interaction of nucleon 1 with the bound nucleon n. In the simple impulse approximation (Chew 1950, Chew and Wick 1952, Queen 1964) the matrix elements of T_n are related to those of t_n, the transition operator for the interaction of two free nucleons 1 and n, by

$$\langle \mathbf{k}', \phi_{\mathbf{q}}, \chi_i, I_1 | T_n | \mathbf{k}, \phi_0, \chi_j, I_0 \rangle \simeq \langle \alpha\mathbf{k}', \chi_i, I_1 | t_n | \tfrac{3}{4}\mathbf{k}, \chi_j, I_0 \rangle S_{\mathbf{q}}\{\tfrac{1}{2}(\mathbf{k}' - \mathbf{k})\} \tag{6}$$

where $S_{\mathbf{q}}$ is an overlap integral defined by

$$S_{\mathbf{q}}(\mathbf{p}) = \int \phi_{\mathbf{q}}^*(\mathbf{r}) \exp(i\mathbf{p} \cdot \mathbf{r}) \phi_0(\mathbf{r})\, d\mathbf{r}. \tag{7}$$

The momenta in the two-body matrix element in (6) refer to the centre-of-mass system of the two interacting nucleons. There is some ambiguity in the choice of the kinematic factor α. The value $\alpha = \frac{3}{4}$, which is suggested by the usual form of the impulse approximation, leads to two-body matrix elements off the energy shell. In order to avoid this unpleasant complication, we choose α instead so that $\alpha k' = \frac{3}{4}k$. This is certainly a good approximation for the high-energy end of the final neutron spectrum, where $k' \simeq k$, although it is somewhat dubious near the low-energy end, where $k' \simeq 0$.

The matrix element in (6) can be related to the conventional representation of the nucleon–nucleon scattering matrix by expanding the three-nucleon spin states in the form

$$\chi_i = \sum_r \lambda_{ir}\eta_r \tag{8}$$

where the η_r ($r = 1, ..., 4$) form an orthonormal set of spin states for the interacting nucleon–nucleon system and the coefficients λ_{ir} involve spin states for the third particle. We choose the η_r, as usual, to be the eigenstates of the square and z component of the spin of the two nucleons. Substituting (2), (3) and (8) into the matrix element of t_n, we obtain, suppressing the momentum states for brevity,

$$\langle \chi_i, I_1|t_n|\chi_j, I_0\rangle = \frac{1}{\sqrt{8}}\sum_{r,s}\langle\lambda_{ir}|\lambda_{js}\rangle(\langle\eta_r|t_n^{(1)}|\eta_s\rangle - \langle\eta_r|t_n^{(0)}|\eta_s\rangle) \tag{9}$$

where $t_n^{(I)}$ is the transition operator for isospin I. The difference of the two isospin amplitudes in (9) is simply twice the nucleon–nucleon charge-exchange amplitude. The complete mathematical formulation of the simple impulse approximation for the d(p, n)2p amplitude in terms of two-body scattering parameters is provided by equations (5), (6) and (9).

It remains only to specify a definite model for the overlap integral (7). For the ground state of the deuteron we take the Hulthén function (Hulthén and Sugawara 1957)

$$\phi_0(\mathbf{r}) = \frac{N(e^{-ar} - e^{-br})}{r} \tag{10}$$

where

$$N^2 = \frac{ab(a+b)}{2\pi(b-a)^2}$$

with the parameter values $a = 0{\cdot}232\ \mathrm{fm}^{-1}$ and $b = 1{\cdot}202\ \mathrm{fm}^{-1}$ (Moravcsik 1958). The wave function (10) is normalized so that

$$\int |\phi_0(\mathbf{r})|^2\, d\mathbf{r} = 1.$$

The corresponding wave function in momentum space is

$$g_0(\mathbf{q}) = \left(\frac{2}{\pi}\right)^{1/2} N\left(\frac{1}{q^2+a^2} - \frac{1}{q^2+b^2}\right). \tag{11}$$

Let us first consider the plane-wave approximation for the continuum states of the final pp system. The wave functions, appropriately normalized and symmetrized to satisfy the Pauli principle, are

$$\phi_{\mathbf{q}}^{(\pm)}(\mathbf{r}) = \frac{\exp(i\mathbf{q}\,.\,\mathbf{r}) \pm \exp(-i\mathbf{q}\,.\,\mathbf{r})}{\sqrt{2}(2\pi)^{3/2}} \tag{12}$$

where the plus (minus) sign is associated with the three-nucleon spin states χ_i which are antisymmetric (symmetric) with respect to interchange of particles 2 and 3. The wave functions (12) yield

$$S_{\mathbf{q}}^{(\pm)}(\mathbf{p}) = \{g_0(\mathbf{p}-\mathbf{q}) \pm g_0(\mathbf{p}+\mathbf{q})\}\frac{1}{\sqrt{2}}. \tag{13}$$

It is desirable, however, to take into account corrections to the plane-wave approximation due to final-state interactions between the two protons. Since we are primarily interested in the high-energy end of the final neutron spectrum, where the two final protons have a small relative energy, only S-wave final-state interactions are expected to be important.

As the S state is absent in the antisymmetric wave functions, the plane-wave approximation (13) is adequate for $S_{\mathbf{q}}^{(-)}$. For the symmetric wave functions, on the other hand, we adopt the approximate form

$$\phi_{\mathbf{q}}^{(+)}(\mathbf{r}) = \frac{1}{\sqrt{2}(2\pi)^{3/2}}\left[\exp(i\mathbf{q}\,.\,\mathbf{r}) + \exp(-i\mathbf{q}\,.\,\mathbf{r}) + 2F(q)\frac{\exp(-iqr)}{r}\{1-\exp(-cr)\}\right] \quad (14)$$

where $F(q)$ is the $^1S_0$ component of the nucleon–nucleon scattering amplitude and c is a cut-off parameter, introduced to avoid a spurious divergence of the wave function at the origin. The cut-off should be effective within the range of the nucleon–nucleon interaction, and we take $c = 1{\cdot}5$ fm^{-1}. The approximate wave function (14), which has been suggested earlier by other authors (Gourdin and Martin 1959), has the correct asymptotic behaviour but is nevertheless sufficiently simple for the overlap integral (7) to be evaluated analytically, if we assume the Hulthén function (10) for the deuteron ground state. Furthermore, the numerical value of the overlap integral is insensitive to the choice of the cut-off parameter c. The amplitude

$$F(q) = \exp\{i\delta(q)\}\sin\{\delta(q)\}\frac{1}{q}$$

where δ is the $^1S_0$ phase shift, is parameterized by the effective-range formula

$$q\cot\delta(q) = -\frac{1}{A} + \tfrac{1}{2}R_0q^2$$

with the parameter values $A = -7{\cdot}826$ fm and $R_0 = 2{\cdot}786$ fm (Noyes 1964).

Substituting the wave functions (10) and (14) into the integral (7), we obtain

$$\begin{aligned} S_{\mathbf{q}}^{(+)}(\mathbf{p}) = {} & \frac{1}{\sqrt{2}}\{g_0(\mathbf{p}-\mathbf{q}) + g_0(\mathbf{p}+\mathbf{q})\} \\ & + \frac{\sqrt{2}\pi NF^*(q)}{(2\pi)^{3/2}p}\left[Q(a) - Q(b) + Q(b+c) - Q(a+c)\right. \\ & \left. + i\log\left\{\frac{R(a)R(b+c)}{R(b)R(a+c)}\right\}\right] \end{aligned}$$

where

$$Q(x) = 2\tan^{-1}\left(\frac{2px}{x^2+q^2-p^2}\right)$$

$$R(x) = \frac{x^2+(p+q)^2}{x^2+(p-q)^2}.$$

Coulomb final-state interactions between the two protons have been neglected here. Although the Coulomb repulsion has a considerable influence on the shape of the energy spectrum of the fast forward neutrons (Phillips 1964), its effect on the depolarization parameter is expected to be small. Near the high-energy end of the neutron spectrum the residual protons are almost entirely in a singlet spin state. In the impulse approximation final-state interactions affect the d(p, n)2p transition amplitude only through the continuum wave functions in the overlap integral (7), thus changing each spin matrix element $\mathscr{M}_{ij}$ by the same factor. Such an overall multiplicative factor has no effect on the polarization of the neutron. Although both singlet and triplet final spin states are important at higher excitation energies of the proton pair, so that effects due to final-state interactions may be significant there, these effects are likely to be dominated by the nuclear interaction.

3. The depolarization parameter

Let us consider the d(p, n)2p reaction for transversely polarized incident protons and unpolarized target deuterons. Let ρ be the spin density matrix of the initial state, normalized so that $\mathrm{Tr}(\rho) = 1$. The transverse polarization of the neutrons in a given final state,

specified by $\mathbf{k}'$ and $\mathbf{q}$, is given by

$$\langle \boldsymbol{\sigma}_1 \rangle_f \cdot \mathbf{n} = \mathrm{Tr}(\mathscr{M}\rho\mathscr{M}^\dagger \boldsymbol{\Sigma}_0 \cdot \mathbf{n}) \frac{1}{J} \tag{15}$$

where $\mathbf{n}$ is the unit vector normal to the scattering plane,

$$J = \mathrm{Tr}\,(\mathscr{M}\rho\mathscr{M}^\dagger)$$

is the scattered intensity corresponding to the initial density matrix ρ and

$$\boldsymbol{\Sigma}_0 = I_{23} \otimes \boldsymbol{\sigma}_1$$

is the direct product of the unit matrix I_{23} in the spin space of particles 2 and 3 with the Pauli spin matrix $\boldsymbol{\sigma}_1$ for particle 1.

The density matrix ρ is given by the direct product

$$\rho = \rho_1 \otimes \rho_{23} \tag{16}$$

where ρ_1 and ρ_{23} are the density matrices for the initial protons and for unpolarized deuterons respectively. The former can be written in terms of the initial proton polarization as

$$\rho_1 = \tfrac{1}{2}(I_1 + \langle \boldsymbol{\sigma}_1 \rangle_i \cdot \boldsymbol{\sigma}_1) \tag{17}$$

where I_1 is the unit matrix in the spin space of nucleon 1. Substituting (16) and (17) into (15), we obtain, for scattering in the forward direction,

$$J\langle \boldsymbol{\sigma}_1 \rangle_f \cdot \mathbf{n} = J_0(P + D\langle \boldsymbol{\sigma}_1 \rangle_i \cdot \mathbf{n}) \tag{18}$$

where

$$J_0 = \mathrm{Tr}(\mathscr{M}\rho_0\mathscr{M}^\dagger)$$

is the scattered intensity for an initially unpolarized state with density matrix ρ_0,

$$P = \mathrm{Tr}(\mathscr{M}\rho_0\mathscr{M}^\dagger \boldsymbol{\Sigma}_0 \cdot \mathbf{n}) \frac{1}{J_0} \tag{19}$$

is the familiar polarization parameter, and

$$D = \mathrm{Tr}(\mathscr{M}\boldsymbol{\Sigma} \cdot \mathbf{n}\mathscr{M}^\dagger \boldsymbol{\Sigma}_0 \cdot \mathbf{n}) \frac{1}{2J_0} \tag{20}$$

where

$$\boldsymbol{\Sigma} = \rho_{23} \otimes \boldsymbol{\sigma}_1.$$

The quantity D is the depolarization parameter which we wish to calculate. For scattering in the forward direction, where $J = J_0$ and $P = 0$, it is evident from (18) that D is indeed the polarization transfer coefficient, i.e.

$$D = \frac{\langle \boldsymbol{\sigma}_1 \rangle_f \cdot \mathbf{n}}{\langle \boldsymbol{\sigma}_1 \rangle_i \cdot \mathbf{n}}. \tag{21}$$

We are particularly interested in the dependence of D on the energy of the neutrons emerging in the forward direction, for a given incident proton energy. However, D is also a function of the final state of the two protons, specified by $\mathbf{q}$. Although the magnitude of $\mathbf{q}$ is fixed by the condition (4), there is no restriction on its direction. Therefore we define an average depolarization parameter

$$\langle D(\mathbf{k}') \rangle = \frac{\int J(\mathbf{k}', \mathbf{q}) D(\mathbf{k}', \mathbf{q})\, d^{(2)}\mathbf{q}}{\int J(\mathbf{k}', \mathbf{q})\, d^{(2)}\mathbf{q}} \tag{22}$$

where the integrations are over the allowed values of $\mathbf{q}$. The quantity $\langle D \rangle$ is the expectation value of D for a given final neutron state and, for $\mathbf{k}'$ in the forward direction, is the value of (21) measured in an experiment in which no regard is paid to the momenta of the two final protons.

Two limiting cases deserve special attention. Let us first consider the high-energy end of the final neutron spectrum, where $\mathbf{q} = 0$. It is obvious from equation (13) that the overlap integral $S^{(-)}$ which occurs in (6) vanishes in this case. If we set $S^{(-)} = 0$ in the equations for the impulse approximation, the general expression (20) for the D parameter reduces to the result obtained by Phillips (1959), assuming that the final proton pair is restricted to the singlet spin state.

The second limiting case of interest is the low-energy end of the final neutron spectrum, where $|\mathbf{q}|$ is large and $\mathbf{k}' = 0$. In this case the overlap integrals associated with the singlet and triplet final spin states are both important and may not differ greatly in magnitude. If we set $|S^{(-)}| = |S^{(+)}|$ in the impulse approximation, expression (20) reduces to the usual formula (Phillips 1959) for the D parameter for free neutron–proton charge-exchange scattering.

4. Multiple-scattering corrections

Multiple-scattering corrections to the impulse approximation can be systematically evaluated with the aid of the formalism of Watson (1953). It is well known that these corrections are rather important for many processes and that, in fact, the impulse approximation may strongly violate elastic unitarity (Queen 1964, 1965, 1966). Moreover, because of the possibility of large cancellations among contributions of various orders, it is desirable to sum certain multiple-scattering terms to all orders.

In an earlier study of elastic nucleon–deuteron scattering (Queen 1964) it was found that the most important multiple-scattering corrections arise from intermediate states on the energy shell, with the deuteron remaining in its ground state. Fortunately, these are the corrections which are most easily taken into account. If off-energy-shell and inelastic intermediate states are neglected, the d(p, n)2p transition matrix is given by (Queen 1965)

$$\langle f|\mathscr{T}|i\rangle = \langle f|T_2|i\rangle + \langle f|T_3|i\rangle - \frac{i\pi\mu}{\hbar^2 k}\sum_l \int \langle f|T_2|\mathbf{k}'', \phi_0, \chi_l, I_0\rangle\langle \mathbf{k}'', \phi_0, \chi_l, I_0|W_3|i\rangle\, d^{(2)}\mathbf{k}'' + (2 \leftrightarrow 3) \quad (23)$$

where i and f denote the initial and final states in the matrix element (1), the integration is over the surface of the sphere $|\mathbf{k}''| = k$, $(2 \leftrightarrow 3)$ denotes a term similar to the preceding one with the roles of particle labels 2 and 3 interchanged, and W_2 and W_3 are solutions of the coupled integral equations

$$\langle n|W_2|i\rangle = \langle n|T_2|i\rangle - \frac{i\pi\mu}{\hbar^2 k}\sum_l \int \langle n|T_2|\mathbf{k}'', \phi_0, \chi_l, I_0\rangle\langle \mathbf{k}'', \phi_0, \chi_l, I_0|W_3|i\rangle\, d^{(2)}\mathbf{k}'' \quad (24)$$

and a similar equation with labels 2 and 3 interchanged, where n denotes one of the intermediate states in equation (23).

Expression (23) includes multiple-scattering corrections of all orders. The quantities W_2 and W_3 which occur here have been defined in connection with elastic scattering by the deuteron in previous papers (Queen 1965, 1966), where the solution of equations (24) has been studied in detail. The determination of the matrix elements of W_2 and W_3 required for equation (23) is, in fact, equivalent to the solution of the elastic nucleon–deuteron scattering problem.

It should be noted that, in the impulse approximation, the matrix elements of T_2 and T_3 occurring in equation (23) are proportional to the difference $t^{(1)} - t^{(0)}$ of the two nucleon–nucleon isospin amplitudes (see equation (9)), whereas the matrix elements required in (24), in which both the initial and final isospin states are I_0, depend only on the combination $\frac{1}{4}t^{(0)} + \frac{3}{4}t^{(1)}$.

5. Numerical calculations

The impulse approximation was used to calculate the average depolarization parameter $\langle D \rangle$ for neutrons emitted in the forward direction in the d(p, n)2p reaction. The calculation was made as a function of the neutron energy E_n in the laboratory system, for values of the incident proton energy E_p equal to 30, 50, 100 and 150 MeV. The required two-body transition matrix elements were evaluated in terms of the Livermore energy-dependent nucleon–nucleon phase shifts (Arndt and MacGregor 1966) at a laboratory energy equal to E_p. The results thus obtained, with and without final-state interactions, are shown in figures 1 and 2.

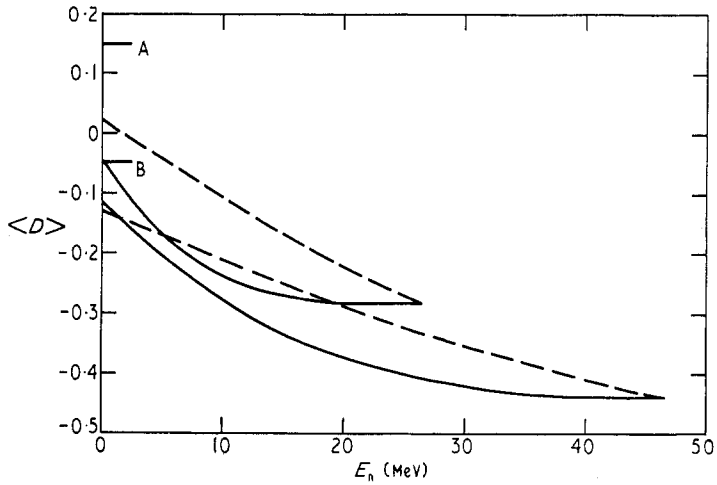

Figure 1. The depolarization parameter $\langle D \rangle$ for neutrons emitted in the forward direction in the d(p, n)2p reaction, as a function of the neutron energy E_n. The broken curves are the impulse-approximation results with the assumption of plane waves for the final two-proton state, and the full curves are the results corrected for final-state interactions. The upper and lower sets of curves are for incident proton energies E_p = 30 MeV and E_p = 50 MeV, respectively. The values of the depolarization parameter D_{np} for free neutron–proton forward charge-exchange scattering at these two energies are indicated by the lines A and B respectively.

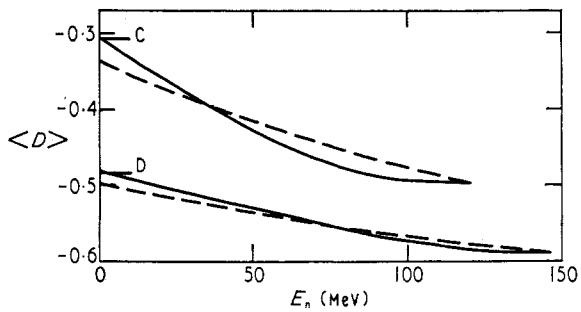

Figure 2. The significance of the curves is the same as in figure 1. The upper and lower sets of curves are for E_p = 100 MeV and E_p = 150 MeV, respectively. The values of D_{np} at these two energies are indicated by the lines C and D respectively.

Final-state interactions do not affect the results at the maximum neutron energy E_n. The reason for this is readily understood in terms of the structure of the impulse approximation, in which each element of the matrix $\mathcal{M}$ is a product of a two-body matrix element and an overlap integral $S^{(\pm)}$. At the high-energy end of the neutron spectrum, where $\mathbf{q} = 0$, the integral $S^{(-)}$ vanishes. Hence the factor $S^{(+)}$, which contains all the dependence

on final-state interactions, cancels out in the formula (20) for the D parameter. On the other hand, the numerical results indicate that final-state interactions are rather important, at least for proton energies up to 50 MeV, for other values of E_n, even for values only slightly below the maximum.

At each of the four energies E_p we also indicate in figure 1 the value of the depolarization parameter D_{np} for free neutron–proton charge-exchange scattering in the forward direction, to be compared with $\langle D \rangle$ at $E_n = 0$. The significant difference between these two quantities at 30 and 50 MeV incident proton energy is accounted for by the fact that, for certain final momentum states of the proton pair, the overlap integrals $S_q^{(+)}$ and $S_q^{(-)}$ in the impulse approximation are significantly different in magnitude. However, as expected, $|S_q^{(+)}|$ and $|S_q^{(-)}|$ approach equality for most final states at higher incident energies, so that $\langle D \rangle$ at $E_n = 0$ tends rapidly to D_{np} as E_p increases.

In the impulse approximation (6) we have adopted the approximation of putting the two-body matrix element on the energy shell, at the energy of the initial state. For energies E_n substantially below the maximum, where the difference between E_n and E_p is large, this approximation is rather arbitrary. We have investigated the sensitivity of the results to the method of restricting the matrix elements to the energy shell, by repeating the calculations with the alternative convention of evaluating these matrix elements at the energy $\frac{1}{2}(E_n+E_p)$. We find that the uncertainty due to this ambiguity is of the same order of magnitude as the correction due to final-state interactions.

Multiple-scattering corrections are included by numerically solving the coupled integral equations (24) and substituting the result into (23). Since our technique of obtaining solutions of (24) has been described elsewhere (Queen 1966), we shall not give the full details here. We merely note that the number of spin matrix elements to be calculated, and hence the number of coupled equations to be solved, can be reduced by exploiting the symmetries of the nucleon–deuteron scattering matrix. Moreover, for reasons justified in the earlier paper, we neglect all multiple-scattering corrections to matrix elements describing transitions between different spin states. This reduces the problem to one of solving only three uncoupled equations.

In spite of the enormous simplifications which we have introduced, the accurate solution of the multiple-scattering equations required a considerable amount of computer time. We have therefore calculated multiple-scattering corrections only for the case $E_p = 50$ MeV. The reliability of the formalism is somewhat doubtful at energies as low as 30 MeV. At the two highest energies, on the other hand, it may be safely assumed that the corrections to the impulse approximation are small.

Table 1. The impulse approximation and multiple-scattering corrections for $\langle D \rangle$ at $E_p = 50$ MeV and $E_n = 46{\cdot}5$ MeV, for four sets of nucleon–nucleon phase shifts

Phase shifts	Impulse approximation	Result corrected for multiple scattering
Livermore (energy-dependent)	−0·436	−0·448
Livermore (energy-independent)	−0·188	−0·186
Harwell	−0·294	−0·299
Yale	−0·166	−0·158

The impulse approximation and the multiple-scattering corrections were evaluated at $E_p = 50$ MeV using several alternative sets of nucleon–nucleon phase shifts near this energy, in order to determine the sensitivity of $\langle D \rangle$ to the phase shifts assumed in the calculation. The sets which we have used are the Livermore energy-dependent and energy-independent phases (Arndt and MacGregor 1966), the Harwell phases (Batty and Perring 1964, 1965) and the Yale phases (Breit *et al.* 1962, Hull *et al.* 1962). The numerical results at the high-energy end of the neutron spectrum ($E_n = 46{\cdot}5$ MeV) are presented in table 1 for these four cases. It is interesting to observe that the value of $\langle D \rangle$ varies from −0·448 for the

Livermore energy-dependent phases to $-0{\cdot}158$ for the Yale phases. The multiple-scattering corrections are surprisingly small in each case shown in the table, but are estimated to be up to three times as large at the low-energy end of the neutron spectrum. Because of the unreliability of the results for small E_n, we do not present detailed numerical results here.

A striking feature of the results in the table is that the sign and magnitude of the multiple-scattering corrections vary considerably from one case to another. This apparently random behaviour is explained by the fact that the multiple-scattering expansion for the transition matrix and the expression for the depolarization parameter in terms of the transition matrix each involves the sum of many positive and negative terms, with large cancellations, so that the results are rather sensitive to small variations in the input data.

Arguments for a new modification of the impulse approximation have recently been advanced by Sloane (1967 a, b). Sloane's method, which includes unitarity corrections to the approximation (6), is equivalent to solving the integral equations (24) with an additional factor of 2 in the kernels. We estimate that the use of Sloane's procedure would lead to corrections to the impulse approximation somewhat larger in magnitude than those which we have calculated, but not large enough to affect any of the conclusions of this paper.

Devins *et al.* (1966) have reported preliminary experimental values for $\langle D \rangle$ at the maximum energy of the forward neutrons emitted in the d(p, n)2p reaction. They obtained $\langle D \rangle = -0{\cdot}16 \pm 0{\cdot}03$ at $E_p = 30$ MeV and $\langle D \rangle = -0{\cdot}26 \pm 0{\cdot}08$ at $E_p = 50$ MeV. It is stressed in their report that their analysis of the data is still incomplete and that these results are therefore tentative. The errors quoted are due to counting statistics only, and there is an additional uncertainty due to the present lack of knowledge of the analysing power of the helium polarimeter used in the experiment. A more precise determination of this analysing power is now in progress (K. Ramavataram, private communication). Although the experimental results so far reported give only qualitative information about $\langle D \rangle$ for forward neutrons in the low-energy region, the data suggest that in this case $\langle D \rangle$ may be positive at $E_p = 30$ MeV, but is small in magnitude at $E_p = 50$ MeV. The experimental results are in qualitative agreement with the theoretical predictions, at least for large E_n, although they appear to be inconsistent with the Livermore energy-dependent phase shifts at 50 MeV.

6. Summary and conclusions

We have studied polarization transfer in the d(p, n)2p reaction as a function of the energy E_n of the neutrons emitted in the forward direction, for incident proton energies E_p between 30 and 150 MeV.

For large E_n with a given E_p the application of the formalism to this process is straightforward and unambiguous. Although corrections to the impulse approximation are small in this case, so that one of the protons may be regarded as a 'spectator' in the reaction, the depolarization parameter is considerably different from the corresponding parameter for free neutron–proton charge-exchange scattering, as a result of the restriction on the final spin state of the proton pair.

For small E_n the depolarization parameter rapidly approaches its value for free neutron–proton charge-exchange scattering as the energy E_p increases. However, in this case the formalism is beset by a number of ambiguities, so that we can only estimate the importance of various effects. The main uncertainty here arises from the application of the simple impulse approximation (6) to the calculation of the single-scattering terms of the d(p, n)2p transition matrix, since the two-body scattering in this process is actually far off the energy shell. In addition, there are significant corrections due to final-state interactions and multiple scattering, although these corrections become less important for large incident proton energies E_p.

At the high-energy end of the neutron spectrum, where our results are most reliable, we have found that the depolarization parameter is negative in the entire energy range studied. If we assume the Livermore energy-dependent phase shifts for the nucleon–nucleon interaction, this parameter varies from $-0{\cdot}284$ at $E_p = 30$ MeV to $-0{\cdot}589$ at

E_p = 150 MeV. We therefore expect that polarization transfer in the d(p, n)2p reaction would be useful as a source of polarized neutrons, particularly at high energies.

The calculations at E_p = 50 MeV, using several alternative sets of nucleon–nucleon phase shifts, indicate that the depolarization parameter at the high-energy end of the final neutron spectrum is particularly sensitive to the phase shifts. An accurate experimental measurement of this parameter would therefore help to eliminate ambiguities in the nucleon–nucleon phase shift analyses.

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